Course C.8-15: Elements of Mathematical Logic

General Overview

This course introduces formalized symbolic logic (mathematical logic) and contrasts it with classical, ontological logic.

Drawing on Alfred Tarski's standard work, the course examines how mathematical logic operates as an abstract calculus of symbols, stripping language of substantive meaning in order to manipulate empty syntactic shells.

Through an analysis of functions, variables and quantifiers, the course explains the structure of propositional logic and class and relation logic, paying particular attention to Philo of Megara's material implication and truth tables.

It then addresses well-known logical problems, such as Leibniz's law of similarity and the liar paradox, contrasting the logical solution through a metalanguage with the suspension of judgment characteristic of natural reasoning.

Finally, the course uncovers the underlying philosophical structure: mathematical logic is grounded in a radical nominalism that reduces reality to humanly constructed conventions and individual signs. When applied to the realities of human life, this produces a cold world devoid of soul or ethical grounding.

Summary

1: Foundations of Mathematical Logic and the Difference from Traditional Logic

The course aims to provide a clear and substantive understanding of formalized symbolic logic - mathematical logic - which, since the late nineteenth century, has been developed by thinkers such as Gottlob Frege and Alfred Tarski as a mode of reasoning for the natural sciences and technology.

The course draws a sharp distinction between traditional (natural) logic and modern mathematical logic. Whereas classical logic is a branch of philosophy concerned with substantive identity, similarity or causal connections between facts in reality, mathematical logic is a mathematical manipulation of symbols (combinatorics).

In mathematical logic, concepts and propositions are stripped of their semantic meaning and reduced to "empty shells" or symbols that can be manipulated according to strictly syntactic and axiomatic-deductive rules.

Although prominent logicians such as Tarski claim that mathematical logic provides the common foundation for all sciences and valid reasoning, Tarski himself acknowledges its clear limits of application.

In empirical practice and everyday human life, where people explore and act within a complex reality, the rigid axiomatic-deductive method proves insufficient on its own, and natural logic remains indispensable.

2: The Logistical Concept of Function, Variables and Quantification

The foundation of the language of mathematical logic rests on the distinction between constants and variables. A constant (such as a specific number or fixed symbol) retains the same meaning throughout an exposition, whereas a variable (such as 'x' or 'y') is a changeable symbol that functions as an empty shell.

When a variable appears in an expression such as "'x' is an integer", this is not yet an assertive proposition in itself, but a propositional function. Only when the variable 'x' is replaced by a constant (for example, the number 1) does a statement with a truth value (true or false) arise. When variables are used to describe objects or numbers mathematically (as in '2x + 1'), the expression is called a descriptive or designative function.

In addition to substituting constants directly, mathematical logic uses quantification to turn a propositional function into a genuine proposition. Quantifiers (operators) are introduced for this purpose:

- *The universal quantifier*: expresses that a function applies to *all* elements ("for all 'x', it holds that…").

- *The existential quantifier*: expresses that a function applies to *some* element or at least one element ("there exists at least one 'x' such that…").

- *The singular quantifier*: concerns *exactly one* specific individual.

By adding these quantifiers, free variables are transformed into bound variables, producing a formally clear and rigorous calculus.

3: Propositional Logic, Material Implication and Truth Tables

In propositional logic (the propositional calculus), unanalyzed sentences are treated as the basic building blocks. Using logical constants or functors, these atomic sentences are combined into molecular propositions:

- *Negation*: reverses the truth value of a sentence.

- *Conjunction* ("and"): is true when both constituent parts are true.

- *Disjunction* ("or"): in mathematical logic, this is consistently used in the non-exclusive sense (inclusive *vel*), meaning that a disjunction is true if at least one of its members is true.

A crucial point of departure from natural reasoning is material implication ("if 'p', then 'q'"), which goes back to the ancient Megarian thinker Philo of

Megara. In natural logic, an "if–then" argument requires a substantive or causal connection between the antecedent and the consequent.

In mathematical logic, this connection is completely severed: material implication is exclusively a combination of truth values. An implication is always 'true' in mathematical logic except when the antecedent is true and the consequent is false. As a result, bizarre statements such as "If 2 × 2 = 5, then New York is a large city" are entirely 'true' in mathematical logic. To ordinary reasoning, this seems absurd because there is no substantive connection between the two statements.

To calculate the truth values of complex compound functions mechanically and systematically, mathematical logic uses truth tables (truth matrices), introduced by Charles Sanders Peirce.

4: Similarity, Class Logic and Relation Logic

The logical concept of 'similarity' or 'identity' ('x = y') rests on Gottfried Wilhelm Leibniz's famous law: two objects are identical if and only if they share all their properties. This relation obeys three fixed laws: reflexivity ('x = x'), symmetry (if 'x = y', then 'y = x') and transitivity (if 'x = y' and 'y = z', then 'x = z').

Unlike ontological logic, which starts from the intrinsic, indivisible identity of a being with itself, mathematical logic reduces identity to a symmetric relation of similarity between separate elements.

In class logic (founded by George Boole and Georg Cantor), individuals are grouped into sets or classes. The statement "'x ∈ K'" means that the individual 'x' belongs to the class 'K'. The course shows that classical logical thought understands concepts in two ways:

- *Distributively* (class): considering all individual instances separately (for example, "all human beings").
- *Collectively* (system): considering the interconnected whole (for example, "all of humanity").

In relation logic (developed by Augustus De Morgan and Charles Peirce), relations between antecedents (the domain) and consequents (the codomain) are calculated formally, as in the relation "'x' is the father of 'y'" or "'x > y'".

5: The Liar Paradox and Philosophical Nominalism

A tried-and-tested litmus test for formal logical systems is the famous liar paradox: the statement “What I am saying now is false.”

Logicians such as Tarski and Józef Maria Bochenski argue that this paradox arises because a language refers to itself. They resolve it by introducing a strict distinction between the object language (which speaks about reality) and the metalanguage (which speaks about the object language), thereby excluding the liar sentence as syntactically invalid or semantically nonsensical.

Natural logic, by contrast, approaches the paradox through suspension of judgment: since both the exact content of “what I am saying now” and the actual intention behind “is false” are unknown, ordinary reasoning simply cannot reach a definitive judgment.

The course concludes by exposing the deepest philosophical axiom underlying mathematical logic: nominalism. Nominalism denies that objective structures of being, essences or intrinsic forms (*eide*) exist in reality. Everything is reduced to arbitrary names, separate symbols and conventions constructed by human beings.

Through John Dewey’s pragmatic instrumentalism and his idea of *social engineering*, this nominalism culminates in the modern dogma that “everything that exists can be engineered”. When this reductive calculus of symbols is applied to the realities of life and the human soul, it produces a cold, mechanical world in which universal ethical norms disappear and are replaced by purely pragmatic and technocratic conventions.